IntComSin Colloquium, 05/12/2023

The stochastic Navier–Stokes–Allen–Cahn system with singular potential

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In this talk, I consider the stochastic version of the Navier–Stokes–Allen–Cahn system in a bounded smooth domain of $\mathbb{R}^d$, with $d \in \{2, 3\}$. The Allen–Cahn system, as proposed by S. M. Allen and J. W. Cahn in the Seventies, is usually seen as a second-order relaxation of the fourth-order Cahn–Hilliard system, yet having some independent interest in a number of contexts (e.g. ordering of atoms during separation phenomena). In order to take hydrodynamical effects into account, a standard coupling with the Navier–Stokes equations is analyzed. Moreover, two independent cylindrical stochastic perturbations, which account for thermodynamical effects (e.g., microscopic collisions) are introduced. The resulting problem is investigated starting from random initial data. A singular potential of logarithmic type, as prescribed by the classical thermodynamical derivation of the model, is considered in the Allen–Cahn system. The problem is endowed with a no-slip conditon for the Navier–Stokes velocity field, as well as homogeneous Neumann conditions for the Allen–Cahn order parameter and chemical potential. During the talk, I will give some insight on the work carried out jointly with Prof. M. Grasselli (Politecnico di Milano) and Dr. L. Scarpa (Politecnico di Milano) towards showing the existence of analytically-weak martingale solutions in two and three spatial dimensions, as well as probabilistically-strong solutions in two dimensions. Open problems and future directions of research will also be presented.