FINITE PROPAGATION SPEED AND DELAY OF SUPPORT PROPAGATION IN STOCHASTIC POROUS MEDIUM EQUATIONS WITH NONLINEAR CONSERVATIVE NOISE

We are concerned with stochastic porous media equations

$$\mathrm{d}u = \Delta(u^m)\,\mathrm{d}t - \operatorname{div}(u^n \circ \mathrm{d}W), \qquad u(0) = u_0, \qquad m > 1.$$

Fehrman & Gess have recently shown the existence of unique non-negative kinetic solutions up to and including the scaling critical exponent $n = (m+1)/2$, provided that the noise is sufficiently colored in space and interpreted in the Stratonovich sense. Based on a localized energy estimate for u, we prove for $1 < n \leq (m+1)/2$ that these kinetic solutions have finite speed of propagation. In particular, we propose a novel iteration technique which allows us to obtain a Stampacchia-type inequality involving one single integral quantity, despite the possibly different scaling behaviors of the porous media and the noise term. This allows us to apply directly the stochastic filtering argument developed first by Fischer & Grün, while still observing the energy production related to the individual operators. Using related ideas, we identify flatness conditions on initial data which guarantee a delay in the onset of forward propagation of the solution's support. The condition for the latter matches the one for $W = 0$ up to a logarithmic correction in the critical case $n = (m+1)/2$, but it requires more and more flatness of the initial data as $n \to 1$. This is in line with the expected behavior of u: For $n = 1$ an instantaneous forward motion is possible due to the effects of stochastic transport, no matter how flat the initial profile is.

The results we present here are part of a joint work with G. Grün (FAU) and M. Sauerbrey (MPI Leipzig).