

Dimension Reduction and Asymptotic Approximation of Reactive Transport in a Bulk Domain with a Branched Thin Fracture

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Thin branched fractures serve as preferential pathways for transport in many heterogeneous media, and understanding the behavior of reactive flow near fracture junctions is essential for predicting macroscopic dynamics. This understanding is crucial in geological formations, biological tissues, and engineered porous or microfluidic systems, where nodal regions strongly influence flow partitioning, reaction efficiency, and signaling.

In this lecture, I present a rigorous multiscale analysis of nonlinear reactive transport in a two-dimensional bulk domain containing a thin branched fracture composed of three narrow branches of thickness ε connected through a junction node of diameter $\mathcal{O}(\varepsilon)$. The microscopic model couples reaction–diffusion in the bulk with advection–diffusion in the fracture, where the longitudinal Péclet number is of order ε^{-1} . As $\varepsilon \rightarrow 0$, the fracture collapses to a one-dimensional graph, and the leading-order behavior is governed by a recurrent structure of limit problems: a first-order hyperbolic equation on the graph with Kirchhoff transmission at the node, coupled with reaction–diffusion equations in the bulk subject to nonlinear Robin conditions involving the graph solution. To capture the influence of the node geometry, we construct node-layer and corner-layer correctors and determine higher-order terms of the asymptotic expansion. The resulting composite approximation yields quantitative error estimates that depend explicitly on the corner angles of the limiting graph, revealing how geometric features of the junction shape the macroscopic transport behavior.