From crystals to Wulff shapes

We introduce the crystallization problem and its connection to
the Wulff problem. We focus on the minimization of Heitman-Radin
potential energies for configurations of N particles. Besides
identifying the asymptotic Wulff shapes through Gamma-convergence, we
introduce the concept of fluctuation and discuss optimal fluctuations
for quasiminimizers proving in particular the well-known $N^{3/4}$
conjecture for minimizers on planar lattices. Our technique combines the
sharp quantitative Wulff inequality with a notion of quantitative
closeness between discrete and continuum problems. We eventually discuss
some recent result on the three dimensional case obtained in
collaboration with Gian Paolo Leonardi (Trento) and Leonard Kreuz (Münster).