On the Navier-Stokes equations under energy-stable outflow boundary conditions

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Many engineering applications involve fluid flow in complex and large domains, making their numerical treatment too computationally expensive or even practically impossible. In order to avoid these implementation drawbacks, domain truncation is a typical procedure in mathematical and numerical modeling. This approach, however, raises the question how to define proper boundary conditions at the artificial boundaries of the computational domain, in order to reproduce the nature of the original physical problem.

In the case of incompressible Navier-Stokes flow, a classical artificial boundary condition is the do-nothing (CDN) condition. Nevertheless, the CDN boundary condition generates stability issues that may compromise existence and computation of solutions for the Navier-Stokes (NS) equations. Consequently, alternative Neumann boundary conditions for the NS equations have been proposed by many authors. C.-H. Bruneau and P. Fabrie introduced a wide class of this type of boundary conditions; S. Dong proposed other energy-stable outflow conditions; and the so-called directional do-nothing (DDN) condition was further developed by M. Braack and P.B. Mucha.

In this seminar, we consider the steady Navier-Stokes equations with mixed boundary conditions, with emphasis on the DDN condition or a regularized directional do-nothing (RDDN) condition on the Neumann portion of the boundary.

Firstly, we present a result on the existence of a weak solution to the NS-DDN system in distorted pipes, which is proved without any smallness assumption on the data, that is, inlet velocity and external force, by combining the Leray-Schauder Principle and the Bernoulli Law.

The DDN boundary condition is defined in terms of a max-function, which is not differentiable in the classical sense. We will discuss the difficulties one faces when trying to apply the Newton method to numerically solve the NS-DDN system and in describing the first order necessary optimality conditions in terms of dual variables when solving a velocity tracking problem for the same system.

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