Singular Stochastic PDEs: How To and What For

(Prof. Dr. Jonas Sauer)

In many models which can be described in terms of differential equations, a lack of knowledge can be compensated by including stochastic noise. Intuitively, this noise captures certain irregular phenomena which are not resolved when looking through the lense of the corresponding deterministic model. The mathematical description of noise reveals a rather fundamental problem when including the noise in SDEs or SPDEs: Typically, realizations of the noise are not smooth functions, but rough distributions. This renders the formulation of many nonlinear differential equations questionable. The theory of rough paths (in the situation of SDEs) and regularity structures (in the situations of SPDEs) is designed to resolve the problems arising from the irregularity of the noise. I will describe the ideas behind these rather modern theories and present some results in the area. Some recent results in the quasilinear case will be discussed as well.