Mathematics of diffuse interface models for multi-phase flows

Andrea Poiatti (Politecnico di Milano)

Phase separation in liquids can be described as a competition between an entropy mixing effect and a demixing effect due to the internal energy. This phenomenon characterizes many important processes, from the behavior of polymer mixtures to biomolecular condensates formation in Cell Biology. Typical mathematical models for phase separation are given by the so-called Cahn-Hilliard (CH) equation or by the conserved Allen-Cahn (CAC) equation with singular potential. These equations govern the evolution of the relative concentration of each component (phase field) and conserve the total mass. Usually more than two chemical species come into play, and this justifies the interest in the extensions of CH equation and CAC equation from binary to multi-component mixtures. In this talk, I will first present the phase separation phenomenon through some of its recent and unexpected applications in Cell Biology. Then I will introduce some of the aforementioned equations in the context of multi-component mixtures, trying to give an insight of the most recent results about the mathematical analysis concerning these equations. In particular, I will discuss the most recent results concerning the validity of the instantaneous strict separation property, which allow to study the longtime behavior of solutions from the point of view of the existence of regular attractors and of the convergence of each trajectory to a single equilibrium. If time permits, I will also present some recent results concerning a diffuse interface model for multi-phase flows of incompressible viscous fluids with different densities (the so-called multi-component Abels-Garcke-Grün model), accounting for well-posedness issues up to the longtime behavior analysis. The works presented in this talk are obtained in collaboration with H. Abels, C.G. Gal, M. Grasselli, H. Garcke, and J.L. Shomberg.