Analysis of a three-dimensional rimming-flow problem

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The talk deals with the modelling and analysis of a capillary-driven three-dimensional rimming-flow problem. We derive a fourth-order quasilinear degenerate-parabolic partial differential equation for the height $h>0$ of a fluid film coating the inner wall of a cylinder that rotates around a horizontal axis.

The equation arises from a rescaled Navier—Stokes system for thin fluid films by means of a lubrication approximation and accounts for the physical effects of rotation, surface tension and gravity. The effect of gravity is indicated by a non-dimensional parameter $0 \leq \delta \ll 1$.

We characterise the structure of the steady states depending on the ratio ℓ of the cylinder length to its radius. In the absence of gravity ($\delta = 0$), in the case $\ell/ \pi \notin \Z$, steady states are unique. For $0 < \delta \ll 1$, steady states are shown to be locally unique for any $\ll$. These steady states are stable for $\ll < \pi$, while they are unstable for $\ell > \pi$.

Furthermore, in the absence of gravity, for all $\ll > 0$, we show that there exists a manifold of time-periodic solutions. In the critical case $\ell = \pi$, we study the dynamics of the solutions close to the manifold of periodic orbits on the large time scale $\tau = \delta^2 t$. It turns out that in the time scale τ this dynamics can be approximated by a system of ODEs.