The Impact of Artificial Intelligence on Partial Differential Equations:
From Successes to Limitations

Artificial intelligence is currently leading to one breakthrough after the other, both in public life with, for instance, autonomous driving and speech recognition, and in the sciences in areas such as medical diagnostics or molecular dynamics. A similarly strong impact can currently be witnessed on scientific computing such as for solvers of partial differential equations.

In this lecture, we will first provide an introduction into this new vibrant research area. We will then specifically focus on high-dimensional parametric partial differential equations (PDEs), which appear in various contexts including control and optimization problems, inverse problems, risk assessment, and uncertainty quantification. Recently, numerical experiments demonstrated the remarkable efficiency of using deep neural networks to solve parametric problems. In the second part of this talk, we will provide both, a theoretical justification for this class of approaches in term of approximation-theoretical results and a comprehensive numerical study. We will finish with a word of caution when training neural networks for solving PDEs on classical digital hardware, and present fundamental limitations.