SOME DISSIPATIVE PROPERTIES OF EULER SOLUTIONS

The incompressible Euler equations describe the conservation of momentum and mass for a perfect fluid. Smooth solutions conserve the total kinetic energy. Since the pioneering works of Kolmogorov and Onsager, a good understanding of turbulence is subject to the study of rough Euler solutions which do not satisfy the exact energy balance, allowing for nontrivial dissipation. Here, we describe fine dissipative properties of bounded weak solutions to the incompressible Euler equations whose first derivatives, or only some combinations of them, are Radon measures. In particular, we obtain elementary proofs of the local energy conservation for solutions in BV and BD. The argument heavily exploits the form of the Euler nonlinearity and it does not apply to the linear transport equations, where the renormalization property for BD vector fields is an open problem. The methods also yields to nontrivial conclusions when only the vorticity is assumed to be a measure.