Homogenization of the Navier-Stokes equations in perforated domains in the inviscid limit

We revisit homogenization problems of fluid flows in perforated domains which have applications in porous media and particulate flows. We consider the solution u_ϵ to the Navier-Stokes equations in $\mathbb{R}^3$ perforated by small particles centered at $(\epsilon\mathbb{Z})^3$ with no-slip boundary conditions at the particles. We study the behavior of u_ϵ for small ϵ, depending on the diameter ϵ^α, $\alpha > 1$, of the particles and the viscosity ϵ^γ, $\gamma > 0$, of the fluid. If the local Reynolds number on the length-scale of the particles is small, one expects that the effective influence of the particles is governed by the asymptotic validity of a linear friction law (Stokeslaw). This reasoning can be made rigorous and effective macroscopic equations are obtained, depending on the order of magnitude of the friction, namely a) the Euler equations in the subcritical regime, b) Darcy's law in the supercritical regimeand c) the Euler-Brinkman equations in the critical regime. The momentum equation of the Euler-Brinkman equations reads

$$\partial_t u + u \cdot \nabla u + \nabla p + \mathcal{R} u = f$$

,where $\mathcal{R} \in \mathbb{R}^{3\times 3}$ is the resistance of there ference particle.