

On the Cauchy Problem for the Quantum hydrodynamic system

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A quantum fluid is a system of interacting particles that exhibits effects of quantum statistics also at a macroscopic scale such as for superfluidity, Bose-Einstein condensation and electron transport in semiconductors. Their prototype model is the Quantum Hydrodynamic system (QHD) describing an inviscid compressible fluid flow featuring a dispersive tensor accounting for the quantum effects. In a broader context, QHD can be viewed as Euler-Korteweg system. Motivated by the study of physically relevant coherent structures, e.g. quantized vortices, we focus on the scenario of non-vanishing density at spatial infinity.

Through the Madelung transform, QHD is formally linked to a class of nonlinear Schrödinger (NLS) equations describing an effective wave function dynamics. First, we discuss well-posedness results for NLS with non-vanishing conditions at infinity. Second, we prove global existence of weak solutions to QHD of finite and infinite energy including vortex-type solutions. These results are based on a rigorous analysis of the analogy of QHD and NLS via a polar factorisation set up in suitable function spaces.

Finally, we review relevant properties of the constructed (vortex) solutions and their link to classical incompressible inviscid vortices in a suitable scaling limit.

Based on joint work with P. Antonelli and P. Marcati (GSSI).