Stability of planar multiphase mean curvature flow beyond circular topology change

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A classical result of Gage–Hamilton and Grayson asserts that any smooth, closed and simple curve in the plane evolving by mean curvature flow (MCF) shrinks to a point in finite time and becomes circular in the process. We establish a weak-strong stability result beyond the time of such circular topology change: any weak (i.e., BV) solution of planar multiphase MCF starting sufficiently close to a smooth, closed and simple curve evolving by MCF has to stay close to it for all times.

Previous weak-strong stability results of this form are limited to time horizons before the first topology change of the strong solution. The reason is that the time-dependent constant in the associated Gronwall estimate is borderline non-integrable. We overcome this issue by developing a weak-strong stability theory for circular topology change up to dynamic shift, which amounts to dynamically adapting the strong solution to the weak solution to a degree which takes care of the leading-order non-integrable contributions in the Gronwall estimate. This strategy is in a sense reminiscent of the recently developed theory of L^2-stability up to shift for solutions of conservation laws close to shock solutions due to Vasseur and co-authors.

This is work in progress with Julian Fischer, Alice Marveggio and Maximilian Moser.