

Mean-field limits for the ionic Vlasov-Poisson system

Dr. Megan Griffin-Pickering (University of Warwick)

Vlasov-Poisson systems are kinetic equations modelling dilute plasma in an electrostatic regime. The precise form of the equation depends on which charged species (electrons or ions) is described: compared to the electron Vlasov-Poisson system, the ionic model includes an additional exponential nonlinearity (representing a thermalised electron background) in the Poisson coupling for the electrostatic potential, which introduces new mathematical challenges.

These equations formally arise as mean-field limits from microscopic systems of charged particles. For electron Vlasov-Poisson, rigorous justification of the mean-field limit has seen significant recent progress, including a proof in 2 dimensions by Duerinckx-Jabin (arXiv:2608.04104). However, in 3 dimensions, the full mean-field limit from point charges is still not fully understood: rigorous results use a truncated Coulomb interaction in order to remove the singularity at small scales. Similar partial results are also available for the ionic equation, but typically require more stringent restrictions on the truncation scale.

In this talk, I will review what is known about the mean-field limit for Vlasov-Poisson equations. Then, I will present a result for the ionic equation, in which the truncation scale is reduced from $1/N^b$ with $b < 1/15$ to $b < 1/3$. The approach is 'probabilistic', in the sense of work on the electron model by Lazarovici-Pickl (ARMA 2017), but the nonlinear, implicit nature of the ionic coupling presents a key new obstacle. I will explain how to overcome this through quantitative law of large numbers estimates for functions.