

Kantorovich duality revisited

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The Kantorovich optimal transport (OT) problem is a convex optimization problem. It is a cornerstone insight in optimization that associated with such a problem there is a dual problem with the same optimal value, and you get very useful information about the primal problem from stating and analyzing the dual. In the case of OT, a prototypical application of duality is a quick modern proof of Brenier's theorem, which I will explain.

Many different versions and proofs of duality in OT exist in the literature starting with Kantorovich 1942, combining abstract Fenchel-Rockafeller (FR) duality with somewhat technical multi-stage approximation arguments for the OT problem. We give a unified, general, and we believe simpler and more transparent treatment which avoids approximation of the OT problem and instead removes the -- seemingly innocent but in practice too restrictive -- requirement in abstract FR duality that either the goal functional or the constraint functional must possess a point of continuity. This new version of FR then applies directly to the natural goal and constraint functionals of OT.