On Symmetric Quasiconvexification and Its Application toward Phase Transformations

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In the theory of linear elastoplasticity, one considers energies of the form

$$\int_{\Omega} F(e(u))\,\mathrm{d}x,$$

which depend on the symmetric gradient $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ rather than on ∇u. To understand the formation of microstructures in composite materials, where non-quasiconvex energy densities F arise naturally, one needs to determine the symmetric quasiconvex envelope of these densities. One strategy for determining this envelope is to construct an upper bound (the symmetric rank-one convex envelope) and a lower bound (the symmetric polyconvex envelope), and then show that they coincide. In this talk, we present an overview of the symmetric notions of semi-convexity and characterize the symmetric polyconvex envelope in three different ways — using a Carathéodory formula, Legendre conjugation, and the translation method. We apply these characterizations to determine the symmetric polyconvex envelope of the function

$$F(\varepsilon) = \min\{(\operatorname{tr}\varepsilon)^2 + \tfrac{1}{2}|\operatorname{dev}\varepsilon|^2 + 1,\ 2(\operatorname{tr}\varepsilon)^2 + |\operatorname{dev}\varepsilon|^2\}$$

in $\mathbb{R}^{2\times 2}_{\mathrm{sym}}$, which describes a phase transformation in an elastic medium. Finally, we show that the resulting function already coincides with the symmetric quasiconvex envelope.