Lack of selection for passive-scalar advection and for the forced Euler equations

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Consider the advection of a passive-scalar under a velocity field in the Hölder class $C^\alpha$. It is known that, in general, for $\alpha<1$ weak solutions may be nonunique. I will show that in the same setting two canonical regularization mechanisms (regularization of the velocity field, and limit of vanishing diffusivity) are also ineffective to select a unique solution in the limit. Moreover, I will show that the same holds for the forced Euler equations in three spatial dimensions. I will also describe the relation to the question of anomalous dissipation for both equations. These results are contained in joint paper with E.~Bru\`e, M.~Colombo, C.~De Lellis, and M.~Sorella.