Error estimates for full discretization of bulk-surface Cahn-Hilliard system with dynamic boundary conditions

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In this talk we present a novel technique for achieving optimal-order error estimates for the full discretization by linear bulk--surface finite elements in space and by the linearly implicit backward difference method of order 1 to 5 in time. The new idea for the stability analysis uses mass conservation properties of the equation to conclude an almost mass conservation of the error, and combines this with a Poincaré-type estimate and an energy estimate. The fully discrete consistency bounds are obtained, by error estimates for the spatial discretization and a sufficiently regular extension of the extrapolation by a Hermite interpolation to obtain error estimates for the backward difference time discretization. This provides optimal-order error estimates between the solution of the numerical scheme and a sufficiently regular exact solution.

We apply this technique to the recently developed bulk--surface Cahn--Hilliard system with general dynamic boundary conditions, which includes a number of important previously known dynamic boundary conditions of Cahn-Hilliard-type, such as the Goldstein-Miranville-Schimperna-model, the Liu-Wu-model and the reaction rate dependent dynamic boundary conditions.

We expect that these techniques can be transferred to other similar equations, satisfying an energy estimate and mass conservation properties, other important examples include Cahn-Hilliard equations on evolving surfaces , or in stationary domains with classical boundary conditions.

Numerical experiments are presented to illustrate and complement the theoretical results.