Designing Structure-Preserving Approximations

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Numerical methods for simulating transient systems that preserve underlying physical or geometric structures often exhibit enhanced accuracy and reliability, particularly over long time intervals. These methods are typically grounded in deep geometric principles and are especially effective for broad classes of problems, such as Hamiltonian or reversible systems.

However, constructing such structure-preserving schemes for general equations remains a significant challenge even when the system's structure is known.

In this talk, we present a framework for designing time discretizations for dissipative Hamiltonian partial differential equations (PDEs), based on continuous Petrov-Galerkin methods in time. We demonstrate the application of these techniques to concrete examples, including Cahn–Hilliard and Navier–Stokes-type systems. Furthermore, we propose compatible spatial discretizations which, in combination with the temporal scheme, yield fully discrete approximations that preserve the intrinsic structure of the original continuous models.